LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – MATHEMATICS & PHYSICS

FOURTH SEMESTER – APRIL 2010

ST 4206 / 4201 - MATHEMATICAL STATISTICS

Date & Time: 19/04/2010 / 9:00 - 12:00 Dept. No. Max. : 100 Marks

PART – A

ANSWER ALL THE QUESTIONS **(10 x 2 = 20 marks)**

1. Define probability function and state the axioms of probability.
2. Define mutual independence of n events.
3. Let X be a random variable with the following probability distribution:

x	-3	6	9
$P(X=x)$	$1/6$	$1/2$	$1/3$

Find E(X) and $E(X^2)$ and using the laws of expectation, evaluate $E(2X+1)^2$.

4. State any two properties of Moment Generating Function.
5. Define discrete uniform distribution and write its mean and variance.
6. Define (i) Beta distribution of first kind and (ii) Gamma distribution.
7. Define F-Statistic and write its probability density function.
8. Define Karl Pearson's coefficient of correlation and write its limits.
9. State invariance property of consistent estimators.
10. Define UMPT.

PART – B

ANSWER ANY FIVE QUESTIONS **(5 x 8 = 40 marks)**

11. (a) For any two events A and B, show that

$$(i)\ P(\bar{A} \cap B) = P(B) - P(A \cap B)$$

$$(ii)\ P(A \cap \bar{B}) = P(A) - P(A \cap B)$$

(b) If $B \subset A,\ then$ show that

$$(i)\ P(A \cap \bar{B}) = P(A) - P(B)$$

$$(ii)\ P(B) \leq P(A)$$

12. Sixty per cent of the employees of the XYZ Corporation are college graduates. Of these, ten percent are in sales. Of the employees who did not graduate from college, eighty per cent are in sales. What is the probability that

    (i) An employee selected at random is in sales?

    (ii) An employee selected at random is neither in sales nor a college graduate?

13. Let X be a continuous random variable with p.d.f.:

$$f(x) = \begin{cases} ax & ,0 \le x \le 1 \\ a & ,1 \le x \le 2 \\ -ax + 3a & ,2 \le x \le 3 \\ 0 & ,elsewhere \end{cases}$$

    (i) Determine the constant a, and

    (ii) Compute $P(X \le 1.5)$

14. Let X and Y be two random variables each taking three values -1, 0, and 1 and having the joint probability distribution

Y \ X	-1	0	1
-1	0	0.1	0.1
0	0.2	0.2	0.2
1	0	0.1	0.1

    (i) Show that X and Y have different expectations.

    (ii) Prove that X and Y are uncorrelated.

    (iii) Find Var X and Var Y.

15. The mean yield for one-acre plot is 662 kilos with a S.D. 32 kilos. Assuming normal distribution, how many one-acre plots in a batch of 1,000 plots would you expect to have yield *(i)* over 700 kilos, *(ii)* below 650 kilos, and *(iii)* what is the lowest yield of the best 100 plots?

16. Define Exponential distribution and derive its moment generating function. Hence obtain its mean and variance.

17. State and prove any two properties of Regression Coefficients.

18. Define the following:

(i) Parameter Space

(ii) Simple and Composite hypotheses

(iii) Level of Significance

(iv) Most Powerful Test

PART – C

ANSWER ANY TWO QUESTIONS **(2 x 20 = 40 marks)**

19 (a) Two dice are rolled. Let X denote the random variable which counts the total number of points on the upturned faces. Construct a table giving the non-zero values of the probability mass function. Also find the distribution function of X.

(b) State and prove the Multiplication theorem of expectation.

(c) If X is the number scored in a throw of a fair die, then show that the Chebychev's inequality gives $P\{|X-\mu| > 2.5\} < 0.47$, where μ is the mean of X, while the actual probability is zero. **(8+4+8)**

20 (a) Derive the M.G.F of Normal distribution and hence obtain its mean and variance.

(b) If X has an exponential distribution, then show that for every constant $a \geq 0$, $P(Y \leq x \mid X \geq a) = P(X \leq x)$ *for all* x, *where* $Y = X - a$.

(c) Define Rectangular distribution and find its mean. **(8+8+4)**

21 (a) The joint probability density function of a two-dimensional random variable (X,Y) is given by

$$f(x,y) = \begin{cases} 2 \;;\; 0 < x < 1,\; 0 < y < x \;; \\ 0 \;,\; elsewhere \end{cases}$$

(i) Find the marginal density functions of X and Y

(ii) Find the conditional density function of Y given X = x and conditional density function of X given Y = y.

(b) Define student's t-statistic and write its p.d.f. Hence obtain the limiting form of t-distribution. **(8+12)**

22 (a) In random sampling from normal population $N(\mu, \sigma^2)$, find the maximum likelihood estimators for

(i) μ when σ^2 is known

(ii) σ^2 when μ is known

(b) If T is an unbiased estimator for θ, then show that T^2 is a biased estimator for θ^2.

(c) State and prove the additional theorem of probability for two events. **(12+4+4)**
